АГ - Г1 - СЕПТ1 2024

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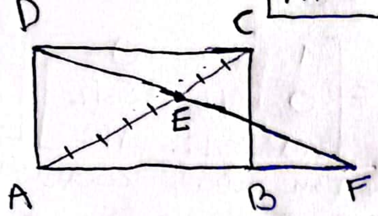

$\vec{AE} = \frac{5}{8}\vec{AC}$. Нека је $\vec{DF} = \alpha \vec{DE}$, $\vec{AF} = \beta \vec{AB}$

$\Rightarrow \vec{DF} = \alpha\vec{DE} = \alpha(\vec{DA}+\vec{AE}) = \alpha\vec{DA} + \alpha\cdot\frac{5}{8}\vec{AC}$

$= -\alpha\vec{AD} + \frac{5}{8}\alpha(\vec{AD}+\vec{DC}) = -\frac{3}{8}\alpha\vec{AD} + \frac{5}{8}\alpha\vec{AB}$

С друге стране, $\vec{DF} = \vec{DA}+\vec{AF} = -\vec{AD}+\beta\vec{AB}$ па изједначавамо

$\Rightarrow \underbrace{\left(1-\frac{3}{8}\alpha\right)}_{=0}\vec{AD} + \underbrace{\left(\frac{5}{8}\alpha-\beta\right)}_{=0}\vec{AB} = 0$, $\vec{AB},\vec{AD}$ лин. нез $\Rightarrow \alpha=\frac{8}{3} \Rightarrow \beta=\frac{5}{3}$

$\Rightarrow \vec{AF} = \frac{5}{3}\vec{AB} = \frac{5}{3}\vec{AF}+\frac{5}{3}\vec{FB} \Rightarrow -\frac{2}{3}\vec{AF} = \frac{5}{3}\vec{FB} \Rightarrow \boxed{\frac{AF}{FB}=\frac{5}{2}}$

2 $P(t,t+1,2t+2)\in p$ произв, $Q(3s+1,-2s+3,3s-1)\in q$ произв.

$r = PQ$ (овим смо обезбедили да $r$ сече праве $p$ и $q$)

1) $r \parallel \alpha \Rightarrow \vec{n}_\alpha \circ \vec{PQ} = 0 \Leftrightarrow (1,1,1)\circ(3s-t+1, -2s-t+2, 3s-2t-3)=0$

$\Rightarrow 3s-t+1-2s-t+2+3s-2t-3=0 \Rightarrow 4s-4t=0 \Rightarrow \boxed{s=t}$

$\Rightarrow \vec{PQ} = (2t+1, -3t+2, t-3)$

2) $d(P,Q)=\sqrt{14} \Rightarrow 14 = \|\vec{PQ}\|^2 = (2t+1)^2+(2-3t)^2+(t-3)^2$ $\quad 14t(t-1)=0$

$\Rightarrow \cancel{14} = 4t^2+4t+\cancel{1}+\cancel{4}-12t+9t^2+t^2-6t+\cancel{9} \Rightarrow 14t^2-14t=0$

$\Rightarrow t_1=0,\ t_2=1 \Rightarrow P_1(0,1,2),\ \vec{P_1Q_1}=(1,2,-3) \vee P_2(1,2,4),\ \vec{P_2Q_2}=(3,-1,-2)$

$\Rightarrow \boxed{r_1 = P_1Q_1: \frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{-3},\quad r_2=P_2Q_2: \frac{x-1}{3}=\frac{y-2}{-1}=\frac{z-4}{-2}}$

3 $A = \begin{bmatrix}15&12&22\\12&8&16\\22&16&20\end{bmatrix}$, $B=\begin{bmatrix}15&12\\12&8\end{bmatrix} \Rightarrow \varphi_B(\lambda) = \begin{vmatrix}15-\lambda & 12\\12 & 8-\lambda\end{vmatrix}$

$= \lambda^2 - 23\lambda - 24 = (\lambda+1)(\lambda-24)$

$\Rightarrow \lambda_1=-1,\ \lambda_2=24$

$\lambda=-1 \Rightarrow \begin{bmatrix}16&12\\12&9\end{bmatrix}\begin{bmatrix}a\\b\end{bmatrix}=0 \Rightarrow 4a+3b=0 \Rightarrow a = -\frac{3}{4}b \Rightarrow v_1 = b\begin{pmatrix}-3/4\\1\end{pmatrix} \Rightarrow v_2\perp v_1 \Rightarrow v_2 \parallel \begin{pmatrix}4\\3\end{pmatrix}$

$\Rightarrow v_1 = \begin{bmatrix}-3/5\\4/5\end{bmatrix} \Rightarrow v_2 = \begin{bmatrix}4/5\\3/5\end{bmatrix}$

$\Rightarrow \begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}-3/5 & 4/5\\4/5&3/5\end{bmatrix}\begin{bmatrix}x'\\y'\end{bmatrix} \Rightarrow \boxed{\begin{aligned}x &= -\tfrac{3}{5}x'+\tfrac{4}{5}y'\\ y&=\tfrac{4}{5}x'+\tfrac{3}{5}y'\end{aligned}}$

$\mathcal{K}': \frac{15}{25}(9x'^2-24x'y'+16y'^2) + \frac{8}{25}(16x'^2+24x'y'+9y'^2)$

$+\frac{24}{25}(-12x'^2+7x'y'+12y'^2) + \frac{44}{5}(-3x'+4y') + \frac{32}{5}(4x'+3y')+20=0$

$\mathcal{K}': -x'^2+24y'^2-\frac{4}{5}x'+\frac{272}{5}y'+20=0$ $\quad \frac{\cancel{272}^{34}}{5\cdot \cancel{24}_3} = \frac{34}{15}$

$\mathcal{K}': -\left(x'+\frac{2}{5}\right)^2+\frac{4}{25}+24\left(y'+\frac{17}{15}\right)^2 - \frac{289}{\cancel{225}_{75}}\cdot\cancel{24}^{8}+20=0$

$\mathcal{K}': 24\underbrace{\left(y'+\frac{17}{15}\right)^2}_{x''} - \underbrace{\left(x'+\frac{2}{5}\right)^2}_{y''} = \frac{2312-1500-12}{75} = \frac{800}{75}=\frac{32}{3}$

$$\boxed{\begin{aligned} x'' &= y' + \tfrac{17}{15} \\ y'' &= x' + \tfrac{2}{5} \end{aligned}} \Rightarrow \mathcal{K}'': 24x''^2 - y''^2 = \frac{32}{3} \quad /: \frac{32}{3}$$

$$\boxed{\mathcal{K}'': \frac{x''^2}{\frac{4}{9}} - \frac{y''^2}{\frac{32}{3}} = 1} \Rightarrow \text{хипербола}$$

$$c = \sqrt{a^2+b^2} = \sqrt{\frac{4}{9} + \frac{96}{9}} = \frac{10}{3} \Rightarrow F_1''\left(\overset{x''}{\frac{10}{3}}, \overset{y''}{0}\right),\ F_2''\left(\overset{x''}{-\frac{10}{3}}, \overset{y''}{0}\right)$$

<u>формуле</u>

$$x = -\frac{3}{5}\left(y'' - \frac{2}{5}\right) + \frac{4}{5}\left(x'' - \frac{17}{15}\right) = \frac{4}{5}x'' - \frac{3}{5}y'' - \frac{2}{3} \Rightarrow \boxed{F_1(2,1)}$$

$$y = \frac{4}{5}\left(y'' - \frac{2}{5}\right) + \frac{3}{5}\left(x'' - \frac{17}{15}\right) = \frac{3}{5}x'' + \frac{4}{5}y'' - 1 \quad \boxed{F_2\left(-\frac{10}{3}, -3\right)}$$

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$\boxed{4}$ $\alpha: 2x - 2y + z + 6 = 0$, $p: \begin{cases} x - 2y + 10 = 0 \\ x + z + 2 = 0 \end{cases} \Rightarrow p: \begin{array}{l} x = 2t - 10 \\ y = t,\ t \in \mathbb{R} \\ z = -2t + 8 \end{array}$

$\Rightarrow \vec{p} = (2, 1, -2)$, $P(-10, 0, 8) \in p$

$M(x, y, z)$ произв $\Rightarrow \mathcal{S}: d(M, \alpha) = d(M, p) = \dfrac{\|\vec{PM} \times \vec{p}\|}{\|\vec{p}\|}$

$$d(M, \alpha) = \frac{|2x - 2y + z + 6|}{\sqrt{4+1+4}} = \frac{|2x - 2y + z + 6|}{3}$$

$$\vec{PM} \times \vec{p} = \begin{vmatrix} i & j & k \\ x+10 & y & z-8 \\ 2 & 1 & -2 \end{vmatrix} = (-2y - z + 8,\ 2z + 2x + 4,\ x - 2y + 10)$$

$$\Rightarrow \mathcal{S}: \frac{|2x - 2y + z + 6|}{\cancel{3}} = \frac{\sqrt{(2y + z - 8)^2 + 4(z + x + 2)^2 + (x - 2y + 10)^2}}{\cancel{\sqrt{4+1+4}}}$$

$$\boxed{\mathcal{S}: (2x - 2y + z + 6)^2 = (2y + z - 8)^2 + 4(x + z + 2)^2 + (x - 2y + 10)^2}$$

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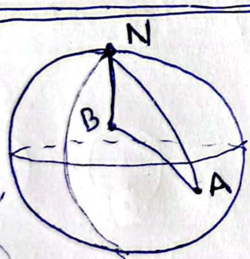


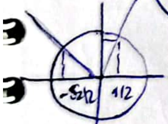


$A: \varphi_A = -\dfrac{\pi}{4}, \theta_A = \dfrac{\pi}{2}$ $\quad B: \varphi_B = \dfrac{\pi}{6}, \theta_B = \dfrac{\pi}{3}$

$\boxed{\text{I начин}}$ прет. да је $R = 1$

$$\cos\overset{\frown}{AB} = \cos\overset{\frown}{NA}\cos\overset{\frown}{NB} + \sin\overset{\frown}{NA}\sin\overset{\frown}{NB}\cos\measuredangle ANB$$

$$\overset{\frown}{NA} = \varphi_N - \varphi_A = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4} \qquad \overset{\frown}{NB} = \varphi_N - \varphi_B = \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}$$

$$\measuredangle ANB = |\theta_A - \theta_B| = \left|\frac{\pi}{2} - \frac{\pi}{3}\right| = \frac{\pi}{6}$$

$$\Rightarrow \cos\overset{\frown}{AB} = -\frac{\sqrt{2}}{2}\cdot\frac{1}{2} + \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{3}}{2} = -\frac{\sqrt{2}}{4} + \frac{3\sqrt{2}}{8} = \frac{\sqrt{2}}{8}$$

за $R = 3390$ $\Rightarrow$ $\boxed{\overset{\frown}{AB} = 3390 \arccos\frac{\sqrt{2}}{8}}$

$\boxed{\text{II начин}}$ прет. да је $R = 1$ $\Rightarrow (x, y, z) = (\cos\varphi\cos\theta, \cos\varphi\sin\theta, \sin\varphi)$

$$\Rightarrow A\left(\frac{\sqrt{2}}{2}\cdot 0, \frac{\sqrt{2}}{2}\cdot 1, -\frac{\sqrt{2}}{2}\right) \Rightarrow A\left(0, \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$$

$$B\left(\frac{\sqrt{3}}{2}\cdot\frac{1}{2}, \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{3}}{2}, \frac{1}{2}\right) \Rightarrow B\left(\frac{\sqrt{3}}{4}, \frac{3}{4}, \frac{1}{2}\right)$$

$$\cos\measuredangle AOB = \vec{OA}\circ\vec{OB} = \frac{3\sqrt{2}}{8} - \frac{\sqrt{2}}{4} = \frac{\sqrt{2}}{8}$$

$$\Rightarrow \boxed{\overset{\frown}{AB} = 3390 \arccos\frac{\sqrt{2}}{8}}$$

(пошто су „лепи" углови можемо радити преко Декартових коорд.)