$\boxed{\text{ЛААГ - септембар 1}}$

$\boxed{1}$ а) ЈАН 2, 2022, 1б б) ЈАН 1, 2022, 1в в) ЈУН 1, 2022, 1в

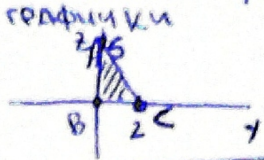
г) $A(0,2,5)$, $Б(0,0,0)$, $C(0,2,0)$

$\vec{BC} = (0,2,0)$, $\vec{BA} = (0,2,5)$

$$\vec{BA} \times \vec{BC} = \begin{vmatrix} i & j & k \\ 0 & 2 & 5 \\ 0 & 2 & 0 \end{vmatrix} = (-10,0,0)$$

$$P_\Delta = \frac{1}{2}\|(-10,0,0)\| = \boxed{5}$$

графички



д) ЈАН 1, 2022, 1г

$\boxed{2}$ $$U = \left\{\begin{bmatrix} c-d & 2c-3d \\ -2c & d \end{bmatrix}\right\} = \left\{\begin{bmatrix} c & 2c \\ -2c & 0 \end{bmatrix} + \begin{bmatrix} -d & -3d \\ 0 & d \end{bmatrix}\right\} = \mathcal{L}\left\{\begin{bmatrix} 1 & 2 \\ -2 & 0 \end{bmatrix}, \begin{bmatrix} -1 & -3 \\ 0 & 1 \end{bmatrix}\right\}$$

$\Rightarrow$ $\dim U = 2$, БАЗА ЗА U је $e_1 = \begin{bmatrix} 1 & 2 \\ -2 & 0 \end{bmatrix}$, $e_2 = \begin{bmatrix} -1 & -3 \\ 0 & 1 \end{bmatrix}$

W: $x + z + t = 0 \Rightarrow x = -z - t$

$y + 3t = 0 \Rightarrow y = -3t$

$$W = \left\{\begin{bmatrix} -z-t & -3t \\ z & t \end{bmatrix}\right\} = \mathcal{L}\left\{\underbrace{\begin{bmatrix} -1 & 0 \\ 1 & 0 \end{bmatrix}}_{f_1}, \underbrace{\begin{bmatrix} -1 & -3 \\ 0 & 1 \end{bmatrix}}_{f_2}\right\}$$

$\Rightarrow$ $\dim W = 2$, БАЗА $f_1, f_2 = e_2$

$$U + W: \begin{array}{c} e_1 \\ e_2 \\ f_1 \\ f_2 \end{array}\begin{bmatrix} 1 & 2 & -2 & 0 \\ -1 & -3 & 0 & 1 \\ -1 & 0 & 1 & 0 \\ -1 & -3 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 0 & -1 & -2 & 1 \\ -1 & -3 & 0 & 1 \\ 0 & 3 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 0 & -1 & -2 & 1 \\ -1 & -3 & 0 & 1 \\ 0 & 0 & -5 & 2 \end{bmatrix}$$

$\boxed{f_2 - e_2 = 0}$

$\Rightarrow$ $\dim U + W = 3$, БАЗА ЗА $U + W$ је e_1, e_2, f_1

$$U \cap W: \underbrace{\dim U + W}_{3} = \underbrace{\dim U}_{2} + \underbrace{\dim W}_{2} - \dim U \cap W$$

БАЗА је e_2

$\Rightarrow$ $\boxed{\dim U \cap W = 1}$

$\boxed{3}$ $L: M_2(\mathbb{R}) \to M_2(\mathbb{R})$, $L(X) = X^T B - \operatorname{tr} X \cdot B$, $B = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

а) 1) $$L(X+Y) = (X+Y)^T B - \operatorname{tr}(X+Y) \cdot B$$
$$= (X^T + Y^T) B - (\operatorname{tr} X + \operatorname{tr} Y) \cdot B$$
$$= X^T B - \operatorname{tr} X \cdot B + Y^T B - \operatorname{tr} Y \cdot B = L(X) + L(Y)$$

2) $$L(\alpha X) = (\alpha X)^T \cdot B - \operatorname{tr}(\alpha X) B = \alpha(X^T B - \operatorname{tr} X \cdot B) = \alpha L(X)$$

$\Rightarrow$ L је линеарно

⑤ $f_1=\begin{bmatrix}1&0\\1&0\end{bmatrix},\ f_2=\begin{bmatrix}2&1\\0&2\end{bmatrix},\ f_3=\begin{bmatrix}3&1\\1&1\end{bmatrix},\ f_4=\begin{bmatrix}0&1\\-1&0\end{bmatrix},\ e$ стандардна база

$$(a,b,c,d)_e = xf_1+yf_2+zf_3+tf_4$$

$$\Rightarrow \begin{cases} x+2y+3z = a \quad (-1) \\ y+z+t=b \\ x+z-t=c \\ 2y+z=d \end{cases}$$

$$\begin{cases} x+2y+3z = a \\ y+z+t=b \quad \cdot 2 \quad \cdot 2 \\ -2y-2z-t=c-a \\ 2y+z=d \end{cases}$$

$$x+2y+3z=a$$
$$y+z+t=b$$
$$\boxed{t=2b+c-a}$$
$$-z-2t=d-2b$$

$$z=2b-d-4b-2c+2a$$
$$\boxed{z=2a-2b-2c-d} \qquad \boxed{y=-a+b+c+d}$$
$$y=b-2a+2b+2c+d-2b-c+a \Rightarrow$$
$$x=a+2a-2b-2c-2d-6a+6b+6c+3d$$
$$\boxed{x=-3a+4b+4c+d}$$

$$\Rightarrow \boxed{(a,b,c,d)_e=(-3a+4b+4c+d,\ -a+b+c+d,\ 2a-2b-2c-d,\ -a+2b+c)_f}$$

$$L(f_1)=\begin{bmatrix}1&1\\0&0\end{bmatrix}\begin{bmatrix}-1&0\\0&-1\end{bmatrix}-\begin{bmatrix}-1&0\\0&-1\end{bmatrix}=\begin{bmatrix}-1&-1\\0&0\end{bmatrix}-\begin{bmatrix}-1&0\\0&-1\end{bmatrix}=\begin{bmatrix}0&-1\\0&1\end{bmatrix}_e=\begin{bmatrix}-3&0\\1&-2\end{bmatrix}_f$$

$$L(f_2)=\begin{bmatrix}2&0\\1&2\end{bmatrix}\begin{bmatrix}-1&0\\0&-1\end{bmatrix}-\begin{bmatrix}-4&0\\0&-4\end{bmatrix}=\begin{bmatrix}-2&0\\-1&-2\end{bmatrix}-\begin{bmatrix}-4&0\\0&-4\end{bmatrix}=\begin{bmatrix}2&0\\-1&2\end{bmatrix}_e=\begin{bmatrix}-8&-1\\4&-3\end{bmatrix}_f$$

$$L(f_3)=\begin{bmatrix}3&1\\1&1\end{bmatrix}\begin{bmatrix}-1&0\\0&-1\end{bmatrix}-\begin{bmatrix}-4&0\\0&-4\end{bmatrix}=\begin{bmatrix}-3&-1\\-1&-1\end{bmatrix}-\begin{bmatrix}-4&0\\0&-4\end{bmatrix}=\begin{bmatrix}1&-1\\-1&3\end{bmatrix}_e=\begin{bmatrix}-8&0\\3&-4\end{bmatrix}_f$$

$$L(f_4)=\begin{bmatrix}0&-1\\1&0\end{bmatrix}\begin{bmatrix}-1&0\\0&-1\end{bmatrix}-\begin{bmatrix}0&0\\0&0\end{bmatrix}=\begin{bmatrix}0&1\\-1&0\end{bmatrix}_e=\begin{bmatrix}0&0\\0&1\end{bmatrix}_f$$

$$\Rightarrow \boxed{[L]_f=\begin{bmatrix}-3&-8&-8&0\\0&-1&0&0\\1&4&3&0\\-2&-3&-4&1\end{bmatrix}}$$

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$$\varphi_A(\lambda)=\det(A-\lambda E)=\begin{vmatrix}-\lambda&1&0&1\\1&-\lambda&1&0\\0&1&-\lambda&1\\1&0&1&-\lambda\end{vmatrix} \begin{matrix}+\\ \cdot\lambda\ (-1)\\ \\ +\end{matrix} = \begin{vmatrix}0&1-\lambda^2&\lambda&1\\1&-\lambda&1&0\\0&1&-\lambda&1\\0&\lambda&0&-\lambda\end{vmatrix}$$

$$=-\begin{vmatrix}1-\lambda^2&\lambda&1\\1&-\lambda&1\\\lambda&0&-\lambda\end{vmatrix}=-\begin{vmatrix}1-\lambda^2&\lambda&2-\lambda^2\\1&-\lambda&2\\\lambda&0&0\end{vmatrix}=-\lambda\begin{vmatrix}\lambda&2-\lambda^2\\-\lambda&2\end{vmatrix}$$

$$=-\lambda\left(2\lambda+\lambda(2-\lambda^2)\right)=-\lambda^2(4-\lambda^2)=\boxed{-\lambda^2(2-\lambda)(2+\lambda)}=\varphi_A(\lambda)$$

$$A(A-2I)(A+2I)=0 \Rightarrow \boxed{\mu_A(\lambda)=\lambda(\lambda-2)(\lambda+2)}$$

$\lambda = 2$ највећа с. вредност

$(A-2I)\,v = 0$

$$\begin{bmatrix} -2 & 1 & 0 & 1 \\ 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \\ 1 & 0 & 1 & -2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = 0 \Rightarrow$$

$$\begin{aligned} -2a + b \quad + d &= 0 \\ a - 2b + c \quad &= 0 \\ b - 2c + d &= 0 \\ a \quad + c - 2d &= 0 \quad \cdot 1 \cdot 2 \\ \hline a \quad + c - 2d &= 0 \Rightarrow a = b \\ b - 2c + d &= 0 \Rightarrow c = b \\ -2b + \quad 2d &= 0 \Rightarrow \boxed{d = b} \\ b + 2c - 3d &= 0 \end{aligned}$$

$$\Rightarrow \boxed{v = b \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \ b \in \mathbb{R}}$$

$\boxed{5}$ $P(2t, -t+3, t+1) \in p$
$Q(s+1, s+2, s) \in q$ $\Rightarrow \overrightarrow{PQ} = (s-2t+1, s+t-1, s-t-1)$

1) $\overrightarrow{PQ} \circ \vec{p} = 0 \Leftrightarrow 2s - 4t + 2 - s - t + 1 + s - t - 1 = 0 \Leftrightarrow \boxed{2s - 6t + 2 = 0}$

2) $\overrightarrow{PQ} \circ \vec{q} = 0 \Leftrightarrow s - 2t + 1 + s + t - 1 + s - t + 1 = 0 \Leftrightarrow \boxed{3s - 2t - 1 = 0} \ (\cdot(-3))$

$\Rightarrow -7s + 5 = 0 \Rightarrow \boxed{s = \frac{5}{7}} \quad \boxed{t = \frac{4}{7}}$

$\Rightarrow P\left(\frac{8}{7}, \frac{17}{7}, \frac{11}{7}\right) \quad Q\left(\frac{12}{7}, \frac{19}{7}, \frac{5}{7}\right) \quad \overrightarrow{PQ} = \left(\frac{4}{7}, \frac{2}{7}, -\frac{6}{7}\right) \parallel \boxed{(2,1,-3)}$

$d(p,q) = \|\overrightarrow{PQ}\| = \sqrt{\frac{16}{49} + \frac{4}{49} + \frac{36}{49}} = \boxed{\frac{\sqrt{56}}{7}}$

$$\boxed{n: \frac{x - \frac{8}{7}}{2} = \frac{y - \frac{17}{7}}{1} = \frac{z - \frac{11}{7}}{-3}}$$

$\boxed{6}$ а) $A^T = -A$

$\det A = \det A^T = \det(-A) = (-1)^n \det A \overset{n \text{ непарно}}{=} -\det A \Rightarrow \det A = 0$

б) Претпоставимо супротно

1) $|A| \neq 0, |B| \neq 0, |C| \neq 0 \Rightarrow 0 = |ABC| = |A| \cdot |B| \cdot |C| \neq 0$ ↯

2) $|A| \neq 0, |B| \neq 0, |C| = 0 \Rightarrow 0 = B^{-1} A^{-1} \cdot 0 = B^{-1} \cdot A^{-1} \cdot ABC = C \Rightarrow C = 0$ ↯
$\Rightarrow \exists A^{-1}, \exists B^{-1}$

3) $|A| \neq 0, |B| = 0, |C| \neq 0 \Rightarrow 0 = A^{-1} \cdot 0 \cdot C^{-1} = A^{-1} ABC C^{-1} = B \Rightarrow B = 0$ ↯
$\Rightarrow \exists A^{-1} \qquad \exists C^{-1}$

4) $|A| = 0, |B| \neq 0, |C| \neq 0 \Rightarrow 0 = 0 C^{-1} B^{-1} = ABC C^{-1} B^{-1} = A \Rightarrow A = 0$ ↯
$\Rightarrow \exists B^{-1}, \exists C^{-1}$