$\boxed{\text{ЈУН 2}}$

$\boxed{1}$ а) Векое, 3.10а

б) $L: U \to V$ је линеарно ако 1) $L(u_1+u_2) = L(u_1)+L(u_2)$, $\forall u_1,u_2 \in U$
2) $(\forall k \in K)$ $L(k \cdot u) = kL(u)$, $\forall u \in U$

$Im(L) = \{v \in V \mid (\exists u \in U)\ L(u) = v\}$, $\dim Im(L) = \rho(L) \to$ ранг

$Ker(L) = \{u \in U \mid L(u) = 0\}$, $\dim Ker L = \delta(L) \to$ дефект

в) Т (Кејли-Хамилтон)

Матрица А поништава свој карактеристични полином, тј. $\varphi_A(A) = 0$.

г) Векое, 8.5

д) $A(27,6) \in p$, $\vec{p} = \vec{AB} = (-7,16) \Rightarrow \boxed{p = AB: \frac{x-27}{-7} = \frac{y-6}{16}}$

ђ) $\vec{a} \cdot \vec{b} = (1,0,1)\cdot(3,2,1) = 3+0+1 = \boxed{4}$

$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 1 & 0 & 1 \\ 3 & 2 & 1 \end{vmatrix} = (-2,2,2)$

$\boxed{2}$ $U = \{(a,b,c,d) \mid a-b-2c=0\} = \{(b+2c, b, c, d) \in \mathbb{R}^4\}$

а) $U \leq \mathbb{R}^4$? 1) $(0,0,0,0) \in U$ јер $\vec{0} = (0+2\cdot 0, 0, 0, 0)$ ✓

2) $u_1 = (b_1+2c_1, b_1, c_1, d_1) \in U$, $u_2 = (b_2+2c_2, b_2, c_2, d_2) \in U$

$\Rightarrow u_1+u_2 = (b_1+b_2+2(c_1+c_2), b_1+b_2, c_1+c_2, d_1+d_2) \in U$ ✓

3) $\alpha \in \mathbb{R}$, $u_1 \in U \Rightarrow \alpha u_1 = (\alpha(b_1+2c_1), \alpha b_1, \alpha c_1, \alpha d_1) \in U$ ✓

$\Rightarrow U \leq \mathbb{R}^4$

$U = \{b\underbrace{(1,1,0,0)}_{e_1} + c\underbrace{(2,0,1,0)}_{e_2} + d\underbrace{(0,0,0,1)}_{e_3} \mid b,c,d \in \mathbb{R}\}$

$\Rightarrow \dim U = 3$, база за U је $\{e_1, e_2, e_3\}$

в) $W = \{(a, 2022, c, 2022) \mid a, c \in \mathbb{R}\}$

нпр

$(x,y,z,w) = (y-2022, y-2022, 0, w-2022) + (x+2022-y, 2022, z, 2022)$

$\Rightarrow \mathbb{R}^4 \subseteq U+W$. Очигледно $U+W \subseteq \mathbb{R}^4 \Rightarrow \boxed{U+W = \mathbb{R}^4}$

$U \cap W = \{(a,b,c,d) \mid a = b+2c, b = d = 2022\}$

$= \{(2022+2c, 2022, c, 2022)\} \neq \{0\} \Rightarrow \mathbb{R}^4 \neq U \oplus W$

$\boxed{3}$ $\varphi_A(\lambda) = -(\lambda-1)(\lambda-2)^2$, $\mu_A(\lambda) = (\lambda-1)(\lambda-2)$

$\lambda_1 = 1 \Rightarrow V_1 = a(-3,1,2)$, $\lambda_2 = 2 \Rightarrow V_2 = b(0,1,0) + c(-1,0,1)$, $b,c \in \mathbb{R}$

$\Rightarrow D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$, $P = \begin{pmatrix} -3 & 0 & -1 \\ 1 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix} \Rightarrow P^{-1} = \begin{pmatrix} -1 & 0 & -1 \\ 1 & 1 & 1 \\ 2 & 0 & 3 \end{pmatrix}$

$A^n = PD^nP^{-1} = \begin{pmatrix} -2\cdot 2^n+3 & 0 & -3\cdot 2^n+3 \\ 2^n-1 & 2^n & 2^n-1 \\ 2\cdot 2^n-2 & 0 & 3\cdot 2^n-2 \end{pmatrix}$

[4] $W=\mathcal{L}(f_1,f_2,f_3,f_4)=\mathcal{L}(f_1,f_2,f_3)$

a) ...

$\Rightarrow e_1=f_1=(1,1,1,1),\quad e_2=(-2,-1,0,3),\quad e_3=(1,1,-3,1)$

$\Rightarrow \hat{e}_1=\left(\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2}\right),\ \hat{e}_2=\left(\frac{-2}{\sqrt{14}},\frac{-1}{\sqrt{14}},0,\frac{3}{\sqrt{14}}\right),\ \hat{e}_3=\left(\frac{1}{\sqrt{12}},\frac{1}{\sqrt{12}},\frac{-3}{\sqrt{12}},\frac{1}{\sqrt{12}}\right)$

б) $W^\perp=\{v \mid v\perp f_1,\ v\perp f_2,\ v\perp f_3\}=\dots=\{d\underbrace{(4,-5,0,1)} \mid d\in\mathbb{R}\}$

$\Rightarrow$ ОНБ за $W^\perp$ је $\hat{g}=\frac{1}{\sqrt{42}}(4,-5,0,1)$

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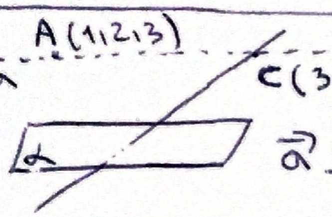
[5] a)



$C(3t+2,2t-4,t+4)$

$A(1,2,3)\in a$

$\vec{a}=\vec{AC}=(3t+1,2t-6,t+1)$

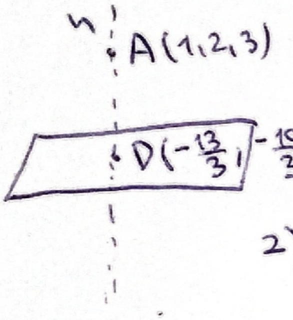
$\vec{a}\perp\vec{n}_\alpha=(1,1,1)\Rightarrow 3t+1+2t-6+t+1=0$

$6t=4\Rightarrow t=\frac{2}{3}$

$\Rightarrow \vec{a}=\left(3,-\frac{14}{3},\frac{5}{3}\right)\parallel(9,-14,5)$

$A(1,2,3)\in a \Rightarrow \boxed{a:\ \frac{x-1}{9}=\frac{y-2}{-14}=\frac{z-3}{5}}$

б)



1) $n\ni A$, $n\perp\alpha$ $\Rightarrow$ $n:\ \begin{array}{l} x=s+1\\ y=s+2\\ z=s+3 \end{array},\ s\in\mathbb{R}$

2) $D=n\cap\alpha:\ s+1+s+2+s+3+10=0$

$3s=-16\Rightarrow s=\frac{-16}{3}$

$A(1,2,3)\longrightarrow D\left(-\frac{13}{3},-\frac{10}{3},-\frac{7}{3}\right)\longrightarrow \boxed{B\left(-\frac{29}{3},-\frac{26}{3},-\frac{23}{3}\right)}$

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[6] вежбе, зад 5.9