ЛААГ - ЈУН 1 - 2022

1. а) ЈАН 2, 1б, 2022 год.
б) вежбе, зад 7.4.
в) вежбе, зад 3.9.
г) вежбе + ЈАН 1, 1г, 2022. год.
д) $U^{\perp} = \{v \in V \mid \langle u, v\rangle = 0,\ \forall u \in U\}$

2.

$$\left[\begin{array}{cccc|cccc} 1 & -1 & 1 & 1 & 1 & 0 & 0 & 0 \\ -1 & 1 & -1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & -1 & 1 & 0 & 0 & 1 & 0 \\ -1 & -1 & -1 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{cccc|cccc} 1 & -1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 1 & 1 & 0 & 0 \\ 0 & 2 & -2 & 0 & -1 & 0 & 1 & 0 \\ 0 & -2 & 0 & 2 & 1 & 0 & 0 & 1 \end{array}\right] \cdot 1/2$$

$$\sim \left[\begin{array}{cccc|cccc} 1 & -1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & -1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 2 & 1 & 1 & 0 & 0 \\ 0 & -2 & 0 & 2 & 1 & 0 & 0 & 1 \end{array}\right] \cdot 2 \sim \left[\begin{array}{cccc|cccc} 1 & -1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & -1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 2 & 1 & 1 & 0 & 0 \\ 0 & 0 & -2 & 2 & 0 & 0 & 1 & 1 \end{array}\right] \begin{array}{l} \\ \\ \cdot 1/2 \\ \cdot -1/2 \end{array}$$

$$\sim \left[\begin{array}{cccc|cccc} 1 & -1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & -1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & -1/2 & -1/2 \\ 0 & 0 & 0 & 1 & 1/2 & 1/2 & 0 & 0 \end{array}\right] (-1) \sim \left[\begin{array}{cccc|cccc} 1 & -1 & 1 & 0 & 1/2 & -1/2 & 0 & 0 \\ 0 & 1 & -1 & 0 & -1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 1 & 0 & 1/2 & 1/2 & -1/2 & -1/2 \\ 0 & 0 & 0 & 1 & 1/2 & 1/2 & 0 & 0 \end{array}\right] +(-1)$$

$$\sim \left[\begin{array}{cccc|cccc} 1 & -1 & 0 & 0 & 0 & -1 & 1/2 & 1/2 \\ 0 & 1 & 0 & 0 & 0 & 1/2 & 0 & -1/2 \\ 0 & 0 & 1 & 0 & 1/2 & 1/2 & -1/2 & -1/2 \\ 0 & 0 & 0 & 1 & 1/2 & 1/2 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 0 & -1/2 & 1/2 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1/2 & 0 & -1/2 \\ 0 & 0 & 1 & 0 & 1/2 & 1/2 & -1/2 & -1/2 \\ 0 & 0 & 0 & 1 & 1/2 & 1/2 & 0 & 0 \end{array}\right]$$

$$\Rightarrow \boxed{A^{-1} = \frac{1}{2}\begin{bmatrix} 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & 1 & 0 & 0 \end{bmatrix}}$$

3. У стандардној бази $M_2(\mathbb{R})$:

U:

$$\begin{array}{c} e_1 \\ e_2 \\ e_3 \end{array}\begin{bmatrix} 2 & 5 & 3 & 1 \\ 3 & 9 & 5 & 1 \\ 2 & 2 & 2 & 2 \end{bmatrix} \begin{array}{l} -3/2\ \ -1 \end{array} \sim \begin{bmatrix} 2 & 5 & 3 & 1 \\ 0 & 3/2 & 1/2 & -1/2 \\ 0 & -3 & -1 & 1 \end{bmatrix} \cdot 2 \sim \begin{bmatrix} 2 & 5 & 3 & 1 \\ 0 & 3/2 & 1/2 & -1/2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \underline{U = \mathcal{L}(e_1, e_2)}$$

$\dim U = 2$

W:

$$\begin{array}{c} f_1 \\ f_2 \\ f_3 \end{array}\begin{bmatrix} 3 & 1 & -1 & 0 \\ -2 & 0 & 2 & 1 \\ 4 & 2 & 0 & 1 \end{bmatrix} \begin{array}{l} 2/3\ \ -4/3 \end{array} \sim \begin{bmatrix} 3 & 1 & -1 & 0 \\ 0 & 2/3 & 4/3 & 1 \\ 0 & 2/3 & 4/3 & 1 \end{bmatrix} -1 \sim \begin{bmatrix} 3 & 1 & -1 & 0 \\ 0 & 2/3 & 4/3 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \underline{W = \mathcal{L}(f_1, f_2)}$$

$\dim W = 2$

$U+W$:

$$\begin{array}{c} e_1 \\ e_2 \\ f_1 \\ f_2 \end{array}\begin{bmatrix} 2 & 5 & 3 & 1 \\ 3 & 9 & 5 & 1 \\ 3 & 1 & -1 & 0 \\ -2 & 0 & 2 & 1 \end{bmatrix} -3/2 \sim \begin{array}{c} e_1 \\ e_2 - 3/2\,e_1 \\ f_1 - 3/2\,e_1 \\ f_2 + e_1 \end{array}\begin{bmatrix} 2 & 5 & 3 & 1 \\ 0 & 3/2 & 1/2 & -1/2 \\ 0 & -13/2 & -11/2 & -3/2 \\ 0 & 5 & 5 & 2 \end{bmatrix} \sim \begin{array}{c} e_1 \\ 2e_2 - 3e_1 \\ 2f_1 - 3e_1 \\ f_2 + e_1 \end{array}\begin{bmatrix} 2 & 5 & 3 & 1 \\ 0 & 3 & 1 & -1 \\ 0 & -13 & -11 & -3 \\ 0 & 5 & 5 & 2 \end{bmatrix}$$

$$\begin{array}{c} e_1 \\ \frac{26}{3}e_2 + 2f_1 - 16e_1 \\ 2e_2 - 3e_1 \\ -\frac{10}{3}e_2 + f_2 + 6e_1 \end{array}\begin{bmatrix} 2 & 5 & 3 & 1 \\ 0 & 3 & 1 & -1 \\ 0 & 0 & -\frac{20}{3} & -22/3 \\ 0 & 0 & 10/3 & 11/3 \end{bmatrix} 1/2$$

$\Rightarrow e_2 + f_1 - 2e_1 + f_2 = 0$

$u = \underline{-e_2 + 2e_1 = f_1 + f_2 \in W}$

$\underline{U + W = \mathcal{L}(e_1, e_2, f_1)}$ $\dim = 3$

$\dim U+W = \dim U + \dim W - \dim U \cap W \Rightarrow 1$

$$\Rightarrow \underline{U \cap W = \mathcal{L}(f_1 + f_2)} = \mathcal{L}((1,1,1,1)) = \mathcal{L}\left(\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\right)$$

4 a)

$$\varphi_A(\lambda)=\begin{vmatrix}-4-\lambda & -1 & 5\\ 3 & 2-\lambda & -3\\ -3 & -1 & 4-\lambda\end{vmatrix} \sim \begin{vmatrix}-4-\lambda & -1 & 1-\lambda\\ 3 & 2-\lambda & 0\\ -3 & -1 & 1-\lambda\end{vmatrix}$$

$$\sim \begin{vmatrix}-1-\lambda & 0 & 0\\ 3 & 2-\lambda & 0\\ -3 & -1 & 1-\lambda\end{vmatrix} = -(\lambda+1)(2-\lambda)(1-\lambda)=\varphi_A(\lambda)$$

$$\Rightarrow \mu_A(\lambda)=(\lambda+1)(\lambda-1)(\lambda-2)$$

$\lambda_1=1$

$$\begin{bmatrix}-5 & -1 & 5\\ 3 & 1 & -3\\ -3 & -1 & 3\end{bmatrix}\begin{bmatrix}a\\ b\\ c\end{bmatrix}=0$$

$-5a-b+5c=0$
$3a+b-3c=0$
$-2a+2c=0$
$\Rightarrow c=a,\ b=0$

$\boxed{v_1=a(1,0,1)}$

$\lambda_2=2$

$$\begin{bmatrix}-6 & -1 & 5\\ 3 & 0 & -3\\ -3 & -1 & 2\end{bmatrix}\begin{bmatrix}a\\ b\\ c\end{bmatrix}=0$$

$\Rightarrow c=a$
$-b-a=0$
$\Rightarrow b=-a$

$\boxed{v_2=a(1,-1,1)}$

$\lambda_3=-1$

$$\begin{bmatrix}-3 & -1 & 5\\ 3 & 3 & -3\\ -3 & -1 & 5\end{bmatrix}\begin{bmatrix}a\\ b\\ c\end{bmatrix}=0$$

$-3a-b+5c=0$
$3a+3b-3c=0$
$2b+2c=0$
$\Rightarrow b=-c,\ a=2c$

$\boxed{v_3=c(2,-1,1)}$

б) $W=\mathcal{L}\{\underset{v_1}{(1,0,1)},\underset{v_2}{(1,-1,1)},\underset{v_3}{(2,-1,1)}\}$,

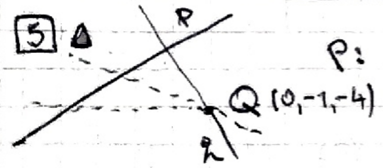
$e_1=v_1=(1,0,1)$

$e_2=v_2-\frac{v_2\circ e_1}{e_1\circ e_1}e_1=(1,-1,1)-\frac{2}{2}(1,0,1)=(0,-1,0)$

$e_3=v_3-\left(\frac{v_3\circ e_1}{e_1\circ e_1}e_1+\frac{v_3\circ e_2}{e_2\circ e_2}e_2\right)=(2,-1,1)-\left(\frac{3}{2},0,\frac{3}{2}\right)-(0,-1,0)=\left(\frac{1}{2},0,-\frac{1}{2}\right)$

ОНБ: $\hat{e}_1=\frac{e_1}{\|e_1\|}=\left(\frac{1}{\sqrt{2}},0,\frac{1}{\sqrt{2}}\right)$, $\hat{e}_2=(0,-1,0)$, $\hat{e}_3=\frac{e_3}{\|e_3\|}=\left(\frac{1}{\sqrt{2}},0,-\frac{1}{\sqrt{2}}\right)$

5



$p:\ x=-3t+17,\ y=t,\ t\in\mathbb{R},\ z=2t-14$

$q=PQ$, где $P(-3t-11,t,2t+14)$

$\vec{q}=\overrightarrow{PQ}=(3t-17,-1-t,-2t+10)\perp\vec{p}(-3,1,2)$

$\Rightarrow -9t+ \quad -1-t-4t+20=0$

$14t=70\Rightarrow \boxed{t=5}\Rightarrow\overrightarrow{PQ}=(-2,-6,0)\parallel(1,3,0)$

$Q(0,-1,-4)\in q$

$\vec{q}=(1,3,0)$

$$\Rightarrow \boxed{q:\ \frac{x}{1}=\frac{y+1}{3}=\frac{z+4}{0}}$$

$Q(0,-1,-4)\in q\subset\alpha$

$p\subset\alpha\Rightarrow\vec{n}_\alpha\perp\vec{p}$

$q\subset\alpha\Rightarrow\vec{n}_\alpha\perp\vec{q}$

$$\Rightarrow \vec{n}_\alpha=\vec{p}\times\vec{q}=\begin{vmatrix}i & j & k\\ -3 & 1 & 2\\ 1 & 3 & 0\end{vmatrix}=(-6,2,-10)\parallel(3,-1,5)$$

$\Rightarrow \alpha:\ 3x-(y+1)+5(z+4)=0$

$$\boxed{\alpha:\ 3x-y+5z+19=0}$$

6

вежбе, зад 5.8.