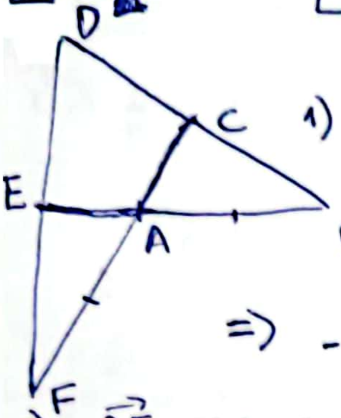
$\boxed{1}$ ▲

$\boxed{\text{АГ - Г1 - ЈУН1 - 2024}}$



$\vec{BD}=2\vec{BC}, \quad \vec{AE}=-\frac{1}{2}\vec{AB}$

1) D, E, F колинеарне $\overset{\text{Менелај}}{\iff}$ $\frac{\vec{BD}}{\vec{DC}}\cdot\frac{\vec{CF}}{\vec{FA}}\cdot\frac{\vec{AE}}{\vec{EB}}=-1$

$\vec{BD}=\vec{BC}+\vec{CD}=\frac{1}{2}\vec{BD}+\vec{CD} \Rightarrow \frac{1}{2}\vec{BD}=\vec{CD}\Rightarrow \boxed{\vec{BD}=-2\vec{DC}}$

$\vec{AE}=\vec{AB}+\vec{BE}=-2\vec{AE}-\vec{EB}\Rightarrow 3\vec{AE}=-\vec{EB}\Rightarrow\boxed{\vec{AE}=-\frac{1}{3}\vec{EB}}$

$\Rightarrow -2\cdot\frac{\vec{CF}}{\vec{FA}}\cdot\frac{-1}{3}=-1 \Rightarrow \boxed{\vec{CF}=-\frac{3}{2}\vec{FA}}$

2) $\vec{AF}$ као лин. комбинац. $\vec{CD}$ и $\vec{BE}$:

$\vec{AC}=\vec{AB}+\vec{BC}\Rightarrow -\frac{1}{2}\vec{AF}=-\frac{2}{3}\vec{BE}+\vec{CD} \quad /\cdot(-2)$

+односи се виде са слике а може и да се срачуна

$\Rightarrow\boxed{\vec{AF}=\frac{4}{3}\vec{BE}-2\vec{CD}}$

3) $P_{\triangle CDF}=\frac{1}{2}\|\vec{CD}\times\vec{CF}\|=\frac{1}{2}\|-\vec{CB}\times 3\vec{CA}\|=3\cdot\overbrace{\frac{1}{2}\|\vec{CB}\times\vec{CA}\|}^{P}=\boxed{3P}$

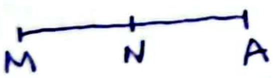
$P_{\triangle AEF}=\frac{1}{2}\|\vec{AE}\times\vec{AF}\|=\frac{1}{2}\|-\frac{1}{2}\vec{AB}\times(-2\vec{AC})\|=\frac{1}{2}\|\vec{AB}\times\vec{AC}\|=\boxed{P}$

$P_{ACDE}=P_{\triangle CDF}-P_{\triangle AEF}=3P-P=\boxed{2P}$

$\boxed{2}$ ▲ Означимо $A(x_1,y_1)$, $B(x_2,y_2)$, $C(x_3,y_3)$

$M(0,-1)$, $N(1,0)$, $P(\frac{3}{2},-\frac{3\sqrt{3}}{2})$, $Q(4,4)$, $R(3,4)$, $O(0,0)$.

1) $A=S_N(M)\Rightarrow \vec{NA}=\underbrace{-\vec{NM}}_{=\vec{MN}}\Rightarrow (x_1-1,y_1)=(1,1)\Rightarrow x_1=2 \quad y_1=1$

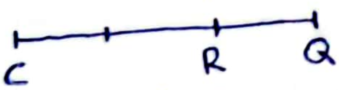


+може и као аритм. низ: $(0,-1)\longrightarrow(1,0)\rightarrow\boxed{A(2,1)}$

2) $B=R_{O,\pi/3}(P)\Rightarrow \begin{bmatrix}x_2\\y_2\end{bmatrix}=\begin{bmatrix}1/2 & -\sqrt{3}/2\\ \sqrt{3}/2 & 1/2\end{bmatrix}\begin{bmatrix}3/2\\ -\frac{3\sqrt{3}}{2}\end{bmatrix}\Rightarrow\boxed{B(3,0)}$

3) $C=\chi_{R,-2}(Q)\Rightarrow \vec{RC}=-2\vec{RQ}\Rightarrow (x_3-3,y_3-4)=-2(1,0)$

$\Rightarrow x_3=1, \ y_3=4 \Rightarrow \boxed{C(1,4)}$



O_{xy}: $A(2,1)$, $B(3,0)$, $C(1,4)$, $\quad O_{x'y'}$: $A(1,0)$, $B(0,1)$, $C(1,1)$

веза координата $\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}a & b\\c & d\end{bmatrix}\begin{bmatrix}x'\\y'\end{bmatrix}+\begin{bmatrix}e\\f\end{bmatrix}$

$A: \begin{bmatrix}2\\1\end{bmatrix}=\begin{bmatrix}a & b\\c & d\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}+\begin{bmatrix}e\\f\end{bmatrix}\Rightarrow a+e=2, \quad c+f=1$

$B: \begin{bmatrix}3\\0\end{bmatrix}=\begin{bmatrix}a & b\\c & d\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}+\begin{bmatrix}e\\f\end{bmatrix}\Rightarrow b+e=3, \quad d+f=0$

$C: \begin{bmatrix}1\\4\end{bmatrix}=\begin{bmatrix}a & b\\c & d\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}+\begin{bmatrix}e\\f\end{bmatrix}\Rightarrow a+b+e=1, \quad c+d+f=4$

$a=2-e, \ b=3-e \Rightarrow 2-e+3-e+e=1\Rightarrow e=4, a=-2, b=-1$

$c=1-f, \ d=-f \Rightarrow 1-f-f+f=4 \Rightarrow f=-3, c=4, d=3$

$\Rightarrow \boxed{\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}-2 & -1\\4 & 3\end{bmatrix}\begin{bmatrix}x'\\y'\end{bmatrix}+\begin{bmatrix}4\\-3\end{bmatrix}}$

3 $\mathcal{K}: 4x^2-4xy+y^2+3x+2y-7=0$

$$A=\begin{bmatrix}4 & -2 & 3/2\\ -2 & 1 & 1\\ 3/2 & 1 & -7\end{bmatrix},\quad B=\begin{bmatrix}4 & -2\\ -2 & 1\end{bmatrix},\quad \varphi_B(\lambda)=\begin{vmatrix}4-\lambda & -2\\ -2 & 1-\lambda\end{vmatrix}=\lambda^2-5\lambda=\lambda(\lambda-5)$$

$\lambda_1=0 \Rightarrow \begin{bmatrix}4 & -2\\ -2 & 1\end{bmatrix}\begin{bmatrix}a\\ b\end{bmatrix}=0 \Rightarrow b=2a \Rightarrow v_1=\begin{pmatrix}1/\sqrt5\\ 2/\sqrt5\end{pmatrix},\ v_2\perp v_1 \Rightarrow v_2=\begin{pmatrix}2/\sqrt5\\ -1/\sqrt5\end{pmatrix}$

$\Rightarrow \boxed{\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}1/\sqrt5 & 2/\sqrt5\\ 2/\sqrt5 & -1/\sqrt5\end{bmatrix}\begin{bmatrix}x'\\ y'\end{bmatrix}} \Rightarrow \boxed{x=\frac{1}{\sqrt5}(x'+2y'),\ y=\frac{1}{\sqrt5}(2x'-y')}$

$\Rightarrow \mathcal{K}': \frac{4}{5}(x'^2+4x'y'+4y'^2)-\frac{4}{5}(2x'^2+3x'y'-2y'^2)+\frac{1}{5}(4x'^2-4x'y'+y'^2)$
$+\frac{3}{\sqrt5}(x'+2y')+\frac{2}{\sqrt5}(2x'-y')-7=0$

$\Rightarrow \mathcal{K}': 5y'^2+\frac{7}{\sqrt5}x'+\frac{4}{\sqrt5}y'-7=0 \Rightarrow 5\left(y'+\frac{2}{5\sqrt5}\right)^2=-\frac{7}{\sqrt5}x'+7+\frac{4}{25}$

$\Rightarrow \mathcal{K}': 5\left(y'+\frac{2}{5\sqrt5}\right)^2=-\frac{7}{\sqrt5}\left(x'-\frac{179}{25}\cdot\frac{\sqrt5}{7}\right)$

$\boxed{x''=x'-\frac{179\sqrt5}{175},\ y''=y'+\frac{2}{5\sqrt5}} \Rightarrow \boxed{\mathcal{K}'': 5y''^2=-\frac{7}{\sqrt5}x''}$ ПАРАБОЛА

Формуле $\boxed{x=\frac{1}{\sqrt5}x'+\frac{2}{\sqrt5}y'=\frac{1}{\sqrt5}x''+\frac{179}{175}+\frac{2}{\sqrt5}y''-\frac{4}{25}=\frac{1}{\sqrt5}x''+\frac{2}{\sqrt5}y''+\frac{151}{175}}$

$\boxed{y=\frac{2}{\sqrt5}x'-\frac{1}{\sqrt5}y'=\frac{2}{\sqrt5}x''+\frac{358}{175}-\frac{1}{\sqrt5}y''+\frac{2}{25}=\frac{2}{\sqrt5}x''-\frac{1}{\sqrt5}y''+\frac{372}{175}}$

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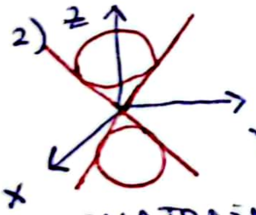
4 1) $p: \begin{array}{l}x=3s+2\\ y=5s-4,\ s\in\mathbb{R}\\ z=-2s+1\end{array}\quad q: \begin{array}{l}x=2t-\lambda\\ y=t+3\\ z=-5,\ t\in\mathbb{R}\end{array} \Rightarrow \begin{array}{l}3s+2=2t-\lambda\\ 5s-4=t+3\\ -2s+1=-5\end{array} \Rightarrow \boxed{\begin{array}{l}\lambda=5\\ t=8\\ s=3\end{array}}$

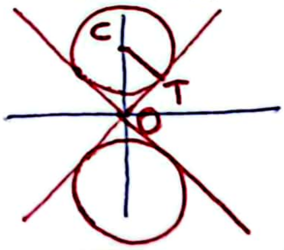
$\Rightarrow \boxed{p\cap q=A(11,11,-5)}$

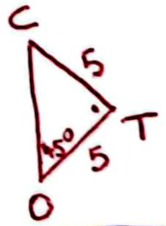
2)



ОСА КОНУСА је Oz-ОСА

ЦЕНТАР C СФЕРЕ је НА ОСИ КОНУСА $\Rightarrow C(0,0,m)$





ПОСМАТРАЈМО ПРЕСЕК У Oyz РАВНИ:

ДОБИЈАМО $\|\overrightarrow{OC}\|=d(O,C)=5\sqrt2$

$\Rightarrow C_1(0,0,5\sqrt2)\quad C_2(0,0,-5\sqrt2)$

$\Rightarrow \boxed{S_1: x^2+y^2+(z-5\sqrt2)^2=25,\ S_2: x^2+y^2+(z+5\sqrt2)^2=25}$

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5 $\alpha=\pi/3,\ \beta=2\pi/3,\ \rho=\pi/6 \Rightarrow \gamma=\pi+\rho-\alpha-\beta \Rightarrow \gamma=\pi/6$

1) $-\cos\alpha=\cos\beta\cos\gamma-\sin\beta\sin\gamma\cos a \Rightarrow -\frac{1}{2}=-\frac{1}{2}\cdot\frac{\sqrt3}{2}-\frac{\sqrt3}{2}\cdot\frac{1}{2}\cos a$

$\Rightarrow \frac{\sqrt3}{4}\cos a=\frac{1}{2}-\frac{\sqrt3}{4}=\frac{2-\sqrt3}{4} \Rightarrow \boxed{a=\arccos\frac{2-\sqrt3}{\sqrt3}}$

2) $-\cos\beta=\cos\alpha\cos\gamma-\sin\alpha\sin\gamma\cos b \Rightarrow \frac{1}{2}=\frac{1}{2}\cdot\frac{\sqrt3}{2}-\frac{\sqrt3}{2}\cdot\frac{1}{2}\cos b$

$\frac{\sqrt3}{4}\cos b=\frac{\sqrt3-2}{4} \Rightarrow \boxed{b=\arccos\frac{\sqrt3-2}{\sqrt3}}$

3) $-\frac{\sqrt3}{2}=\frac{1}{2}\left(-\frac{1}{2}\right)-\frac{\sqrt3}{2}\cdot\frac{\sqrt3}{2}\cos c \Rightarrow \boxed{c=\arccos\frac{2\sqrt3-1}{3}}$