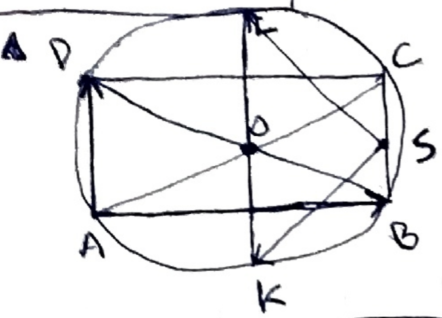
АГ - Г1 - СЕПТ 2 - 2023

1 ABCD правоуг, AB = 2, AD = 1. k описан круг око ABCD. $s_{AB} \cap k = \{K, L\}$. Ae са $\vec{e_1} = \vec{AB}$, $\vec{e_2} = \vec{AD}$ и репер S_f дат са $S = S(BC)$, $\vec{f_1} = \vec{SK}$, $\vec{f_2} = \vec{SL}$. Формуле трансф. координата?



$KL \parallel AD \Rightarrow \vec{OL}, \vec{OK}$ су истог правца као јед. вектор $\vec{AD}$

$$[S]_{Ae} = [\vec{AS}]_e = [\vec{AB} + \vec{BS}]_e = [e_1 + \tfrac{1}{2} e_2] = \boxed{(1, 1/2)}$$

$$[\vec{f_1}]_e = [\vec{SK}]_e = [\vec{SO} + \vec{OK}]_e = [-\tfrac{1}{2}\vec{AB} - \|\vec{OK}\| \cdot \vec{AD}]_e$$

$$= [-\tfrac{1}{2}\vec{e_1} - \tfrac{\sqrt5}{2}\vec{e_2}] = \boxed{\left(-\tfrac{1}{2}, -\tfrac{\sqrt5}{2}\right)}$$ јер је $\|\vec{OK}\| = \frac{R}{2} = \frac{\sqrt5}{2}$

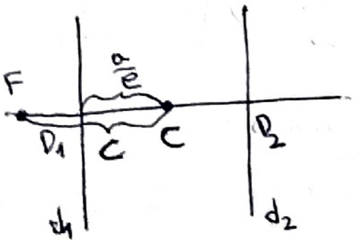
Аналогно, $[\vec{f_2}]_e = [\vec{SO} + \vec{OL}] = \dots = \boxed{\left(-\frac{1}{2}, \frac{\sqrt5}{2}\right)}$

$$\Rightarrow \boxed{\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1/2 & -1/2 \\ -\sqrt5/2 & \sqrt5/2 \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix} + \begin{bmatrix} 1 \\ 1/2 \end{bmatrix}}$$ или $$\boxed{\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & -1/\sqrt5 \\ -1 & 1/\sqrt5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 + 1/2\sqrt5 \\ 1 - 1/2\sqrt5 \end{bmatrix}}$$

2 Крива II реда са директр. $d_1: x + y - 2 = 0$, $d_2: 3x + 3y - 4 = 0$ и жижом $F(2,1)$

$\mathcal{K}: \dfrac{d(M,F)}{d(M,d)} = e$, остаје да одредимо e као и којој директриси одговара жижа $F(2,1)$

$$d(F, d_1) = \frac{|2+1-2|}{\sqrt{1+1}} = \frac{1}{\sqrt2} = \frac{3}{3\sqrt2} < d(M, d_2) = \frac{|6+3-4|}{\sqrt{9+9}} = \frac{5}{3\sqrt2} \Rightarrow (F, d_1) \text{ је пар}$$



Тражимо e: за то нам треба и оса σ

$\sigma \perp d_1, F \in \sigma \Rightarrow \sigma: \frac{x-2}{1} = \frac{y-1}{1} \Rightarrow \boxed{\sigma: x - y - 1 = 0}$

$D_1 = \sigma \cap d_1: \begin{cases} x - y - 1 = 0 \\ x + y - 2 = 0 \end{cases} \Rightarrow D_1\left(\frac{3}{2}, \frac{1}{2}\right)$

$D_2 = \sigma \cap d_2: \begin{cases} x - y - 1 = 0 \\ 3x + 3y - 4 = 0 \end{cases} \Rightarrow D_2\left(\frac{7}{6}, \frac{1}{6}\right)$

$\Rightarrow \boxed{C = S(D_1, D_2) = \left(\frac{4}{3}, \frac{1}{3}\right)}$

(*) $\dfrac{a}{e} = d(C, d_1) = \dfrac{|\frac{4}{3} + \frac{1}{3} - 2|}{\sqrt2} = \dfrac{1}{3\sqrt2} \Rightarrow \boxed{e = 3\sqrt2\, a}$ $\quad \boxed{e = \frac{c}{a}}$

(*) $c = d(C, F) = \sqrt{\left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^2} = \boxed{\frac{2\sqrt2}{3} = c}$

$3\sqrt2\, a = \dfrac{2\sqrt2}{3a} \Rightarrow 9a^2 = 2$
$\Rightarrow a = \dfrac{\sqrt2}{3}$

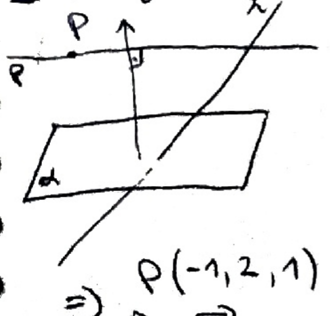
$\Rightarrow e = \dfrac{c}{a} = \dfrac{\frac{2\sqrt2}{3}}{\frac{\sqrt2}{3}} = 2 \Rightarrow \boxed{e = 2}$

$$\Rightarrow \mathcal{K}: 2 = e = \frac{d(M,F)}{d(M,d_1)} = \frac{\sqrt{(x-2)^2 + (y-1)^2}}{\frac{|x+y-2|}{\sqrt2}}$$

$\Rightarrow \boxed{\mathcal{K}: (x-2)^2 + (y-1)^2 = 2(x+y-2)^2}$, $e = 2 > 1 \Rightarrow$ хипербола

3 p т.д. $P(-1,2,1) \in p$, p сече $q: \frac{x-1}{2} = \frac{y+2}{2} = \frac{z+2}{3}$, $p \parallel \alpha: 2x + 3y + z + 2023 = 0$

урађено на три начина на вежбама, ево једног:



$Q(2t+1, 2t-2, 3t-2) \in q$ произвољна

$p = PQ$ за Q т.д. $\vec{PQ} = (2t+2, 2t-4, 3t-3) \parallel \alpha$, $\vec{n}_\alpha = (2,3,1)$

$\Rightarrow \vec{PQ} \circ \vec{n}_\alpha = 0 \Leftrightarrow 4t + 4 + 6t - 12 + 3t - 3 = 0 \Rightarrow 13t = 11 \Rightarrow t = \frac{11}{13}$

$\Rightarrow \vec{PQ} = \left(\frac{48}{13}, \frac{-30}{13}, \frac{-6}{13}\right) \parallel \boxed{(8, -5, -1)}$

$\Rightarrow \begin{rcases} P(-1,2,1) \in p \\ \vec{p} = \vec{PQ} \parallel (8,-5,-1) \end{rcases} \Rightarrow \boxed{p: \dfrac{x+1}{8} = \dfrac{y-2}{-5} = \dfrac{z-1}{-1}}$

4 $\rho: 2x^2 + 2z^2 - 2xz + 2x + 3y + 2z + 2 = 0$

$$A = \begin{bmatrix} 2 & 0 & -1 & 1 \\ 0 & 0 & 0 & 3/2 \\ -1 & 0 & 2 & 1 \\ 1 & 3/2 & 1 & 2 \end{bmatrix} \Rightarrow \varphi_B(\lambda) = \begin{vmatrix} 2-\lambda & 0 & -1 \\ 0 & -\lambda & 0 \\ -1 & 0 & 2-\lambda \end{vmatrix} = -\lambda\left[(2-\lambda)^2 - 1\right] = \underline{-\lambda(\lambda-1)(\lambda-3)}$$

$\lambda_1 = 1 \Rightarrow \vec{v}_1 = \left(\frac{\sqrt2}{2}, 0, \frac{\sqrt2}{2}\right)$, $\lambda_2 = 0 \Rightarrow \vec{v}_2 = (0,1,0)$, $\lambda_3 = 3 \Rightarrow \vec{v}_3 = \left(-\frac{\sqrt2}{2}, 0, \frac{\sqrt2}{2}\right)$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \sqrt2/2 & 0 & -\sqrt2/2 \\ 0 & 1 & 0 \\ \sqrt2/2 & 0 & \sqrt2/2 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} \Rightarrow \boxed{\begin{aligned} x &= \tfrac{\sqrt2}{2}(x'-z') \\ y &= y' \\ z &= \tfrac{\sqrt2}{2}(x'+z') \end{aligned}}$$

$\Rightarrow \rho': x'^2 - \cancel{2x'z'} + z'^2 + x'^2 + \cancel{2x'z'} + z'^2 - x'^2 + z'^2 + \sqrt2(x'-z') + 3y' + \sqrt2(x'+z') + 2 = 0$

$\rho': x'^2 + 3z'^2 + 2\sqrt2 x' + 3y' + 2 = 0 \Leftrightarrow \rho': \underbrace{(x'+\sqrt2)}_{x''}{}^2 + 3\underbrace{z'}_{z''}{}^2 = \underbrace{-3y'}_{\text{"}y''\text{"}}$

$$\Rightarrow \boxed{f: \begin{aligned} x &= \tfrac{\sqrt2}{2}(x'' - \sqrt2 - z'') \\ y &= -y'' \\ z &= \tfrac{\sqrt2}{2}(x'' - \sqrt2 + z'') \end{aligned}} \qquad \Rightarrow \boxed{\rho'': x''^2 + 3z''^2 = 3y''}$$

елиптички параболоид

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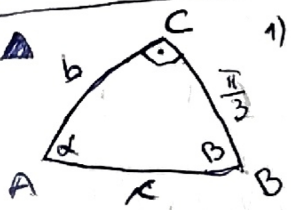
5 $A\left(\frac{\sqrt2}{4}, \frac{\sqrt2}{4}, \frac{\sqrt3}{2}\right)$, B, $C(0,0,1)$ т.д. ΔABC прав. $\measuredangle BCA = \frac{\pi}{2}$ и $\overset{\frown}{BC} = \pi/3 \Rightarrow P = ?$



1) $\cos b = \cos \overset{\frown}{AC} = \cos \measuredangle(\vec{OA}, \vec{OC}) = \dfrac{\vec{OA} \circ \vec{OC}}{\|\vec{OA}\| \, \|\vec{OC}\|} = \dfrac{\sqrt3}{2} \Rightarrow \boxed{b = \frac{\pi}{6}}$

2) Косинусна (Т) $\Rightarrow \cos c = \cos a \cos b + \sin a \sin b \cancel{\cos \gamma}^{\,0}$

$\Rightarrow \cos c = \cos\frac{\pi}{3}\cos\frac{\pi}{6} = \frac{\sqrt3}{4} \Rightarrow \boxed{c = \arccos\frac{\sqrt3}{4}}$

Имамо све три странице, фале углови α и β:

3) Синусна (Т) $\Rightarrow \dfrac{\sin a}{\sin \alpha} = \dfrac{\sin c}{\sin \gamma} \Rightarrow \sin\alpha = \dfrac{\sin a \cdot \sin\gamma}{\sin c} = \dfrac{\frac{\sqrt3}{2} \cdot 1}{\frac{\sqrt{13}}{4}} = \dfrac{2\sqrt3}{\sqrt{13}}$

$\sin^2 c = 1 - \cos^2 c = 1 - \frac{3}{16} = \frac{13}{16} \Rightarrow \sin c = \frac{\sqrt{13}}{4}$ $\qquad \Rightarrow \boxed{\alpha = \arcsin\frac{2\sqrt3}{\sqrt{13}}}$

4) Синусна (Т) $\dfrac{\sin b}{\sin \beta} = \dfrac{\sin c}{\sin\gamma} \Rightarrow \sin\beta = \dfrac{\sin b \sin \gamma}{\sin c} = \dfrac{\frac12 \cdot 1}{\frac{\sqrt{13}}{4}} = \dfrac{2}{\sqrt{13}}$

$\Rightarrow \boxed{\beta = \arcsin\frac{2}{\sqrt{13}}}$

$$\boxed{P = \alpha + \beta + \gamma - \pi = \arcsin\frac{2\sqrt3}{\sqrt{13}} + \arcsin\frac{2}{\sqrt{13}} - \frac{\pi}{2}}$$