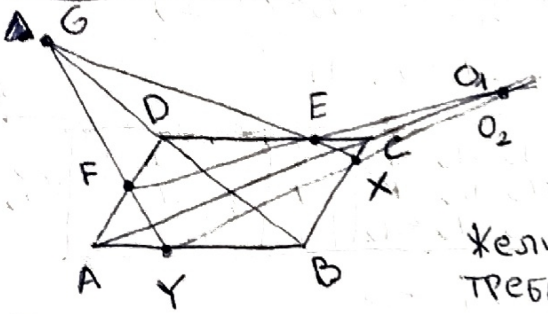
1 ABCD паралелограм, E, F и G т.д. $\overrightarrow{AF} = \overrightarrow{FD}$, $\overrightarrow{DE} = 3\overrightarrow{EC}$, $\overrightarrow{BG} = 2\overrightarrow{BD}$. Ако је $X = CB \cap GE$, $Y = AB \cap GF$ доказати да су AC, EF и XY конкурентне.



Нека је $EF \cap AC = O_1$ и $YX \cap AC = O_2$. Доказујемо да је $O_1 \equiv O_2$:

ΔACD, F, E, O_1 колин. $\xRightarrow{\text{Менелај}}$ $\dfrac{\overrightarrow{AO_1}}{\overrightarrow{O_1C}} \cdot \underbrace{\dfrac{\overrightarrow{CE}}{\overrightarrow{ED}}}_{1/3} \cdot \underbrace{\dfrac{\overrightarrow{DF}}{\overrightarrow{FA}}}_{1} = -1 \Rightarrow \boxed{\dfrac{\overrightarrow{AO_1}}{\overrightarrow{O_1C}} = -3}$ (*)

Желимо исто за ΔACB и Y, X, O_2. За то нам треба у ком односу X и Y деле BC и AB

I ΔABD, Y, F, G колин $\xRightarrow{\text{Мен}}$ $\dfrac{\overrightarrow{AY}}{\overrightarrow{YB}} \cdot \underbrace{\dfrac{\overrightarrow{BG}}{\overrightarrow{GD}}}_{-2} \cdot \underbrace{\dfrac{\overrightarrow{DF}}{\overrightarrow{FA}}}_{1} = -1$

$\Rightarrow \boxed{\dfrac{\overrightarrow{AY}}{\overrightarrow{YB}} = \dfrac{1}{2}}$

II ΔBDC, G, E, X колин $\xRightarrow{\text{Мен}}$ $\dfrac{\overrightarrow{BX}}{\overrightarrow{XC}} \cdot \underbrace{\dfrac{\overrightarrow{CE}}{\overrightarrow{ED}}}_{1/3} \cdot \underbrace{\dfrac{\overrightarrow{DG}}{\overrightarrow{GB}}}_{-1/2} = -1$

$\Rightarrow \boxed{\dfrac{\overrightarrow{BX}}{\overrightarrow{XC}} = 6}$

$\Downarrow$

ΔACB, Y, X, O_2 колин $\xRightarrow{\text{Мен}}$ $\dfrac{\overrightarrow{AO_2}}{\overrightarrow{O_2C}} \cdot \underbrace{\dfrac{\overrightarrow{CX}}{\overrightarrow{XB}}}_{1/6} \cdot \underbrace{\dfrac{\overrightarrow{BY}}{\overrightarrow{YA}}}_{2} = -1$

$\Rightarrow \boxed{\dfrac{\overrightarrow{AO_2}}{\overrightarrow{O_2C}} = -3}$ (**)

(*), (**) $\Rightarrow O_1 \equiv O_2 \Rightarrow$ AC, EF и XY конкурентне

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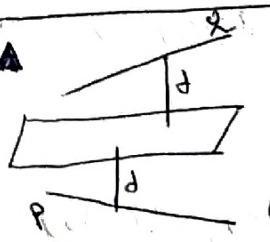
2 $p: \dfrac{x-1}{1} = \dfrac{y+2}{2} = \dfrac{z+2}{2}$, $q: \dfrac{x-2}{3} = \dfrac{y}{-2} = \dfrac{z+2}{2}$. Наћи α подједнако удаљену од p и q.



I $\begin{matrix}\alpha \parallel p \\ \alpha \parallel q\end{matrix} \Rightarrow \vec{n}_\alpha = \vec{p} \times \vec{q} = \begin{vmatrix} i & j & k \\ 1 & 2 & 2 \\ 3 & -2 & 2 \end{vmatrix} = (8, 4, -8) \parallel (2, 1, -2)$

$\Rightarrow \underline{\alpha: 2x + y - 2z + D = 0}$

$P(1, -2, -2) \in p$
$Q(2, 0, -2) \in q$

II $d(p, \alpha) = d(q, \alpha) \Leftrightarrow d(P, \alpha) = d(Q, \alpha)$

$\Leftrightarrow \dfrac{|2 - 2 + 4 + D|}{\sqrt{9}} = \dfrac{|4 + 0 + 4 + D|}{\sqrt{9}} \Rightarrow |4 + D| = |8 + D|$

$4 + D = 8 + D$ ↯

$4 + D = -8 - D$
$2D = -12 \Rightarrow \boxed{D = -6}$

$\Rightarrow \boxed{\alpha: 2x + y - 2z - 6 = 0}$

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3 Све параболе с директр. $d: x + y + 1 = 0$ и садрже $A(3,0)$ и $B(7,0)$.

$d: x + y + 1 = 0$ директриса, нека је $F(\alpha, \beta)$ жижа, $M(x, y)$ произв. тачка параболе

$\Rightarrow \mathcal{P}: d(M, F) = d(M, d) \Rightarrow \mathcal{P}: \sqrt{(x-\alpha)^2 + (y-\beta)^2} = \dfrac{|x + y + 1|}{\sqrt{1^2 + 1^2}} \quad /^2$

$\Rightarrow \underline{\mathcal{P}: 2(x-\alpha)^2 + 2(y-\beta)^2 = (x + y + 1)^2}$ → јна произв. параболе с директрисом d

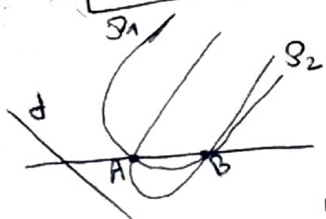
① $A(3,0) \in \mathcal{P} \Rightarrow 2(3-\alpha)^2 + 2\beta^2 = 16 \xRightarrow{:2} \alpha^2 - 6\alpha + \beta^2 = -1$

② $B(7,0) \in \mathcal{P} \Rightarrow 2(7-\alpha)^2 + 2\beta^2 = 64 \xRightarrow{:2} \alpha^2 - 14\alpha + \beta^2 = -17 \quad / \cdot (-1)$

$\Rightarrow 8\alpha = 16 \quad \boxed{\alpha = 2}$

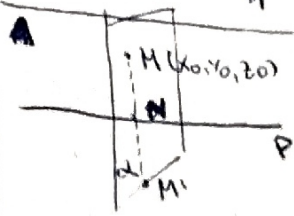
$\Rightarrow \beta^2 = -1 - 4 + 12 = 7 \Rightarrow \boxed{\beta = \pm\sqrt{7}}$

$$\Rightarrow \boxed{\begin{aligned} \mathcal{P}_1 &: 2(x-2)^2 + 2(y-\sqrt{7})^2 = (x+y+1)^2 \\ \mathcal{P}_2 &: 2(x-2)^2 + 2(y+\sqrt{7})^2 = (x+y+1)^2 \end{aligned}}$$



4 $\sigma_P,\ P: \frac{x}{1}=\frac{y-1}{2}=\frac{z}{1},\ \chi_{S,2},\ S(1,3,1) \Rightarrow \sigma_P\circ\chi_{S,2}=?$

I



1) α т.д. $M\in\alpha,\ \alpha\perp P$

$\alpha: 1(x-x_0)+2(y-y_0)+z-z_0=0$

$\alpha: x+2y+z-x_0-2y_0-z_0=0$

2) $N=P\cap\alpha$, $P: \begin{array}{l} x=t \\ y=2t+1 \\ z=t \end{array},\ t\in\mathbb{R}$

$\Rightarrow t+4t+2+t-x_0-2y_0-z_0=0$

$\boxed{t=\frac{1}{6}(x_0+2y_0+z_0-2)}$

$\Rightarrow P\cap\alpha = \boxed{N\left(\frac{1}{6}(x_0+2y_0+z_0-2),\ \frac{1}{3}(x_0+2y_0+z_0+1),\ \frac{1}{6}(x_0+2y_0+z_0-2)\right)}$

3) $[N]=\frac{[M]+[M']}{2} \Rightarrow [M']=2[N]-[M] \Rightarrow \boxed{\begin{bmatrix}x'\\y'\\z'\end{bmatrix}=\begin{bmatrix}-2/3 & 2/3 & 1/3\\ 2/3 & 1/3 & 2/3\\ 1/3 & 2/3 & -2/3\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}+\begin{bmatrix}-2/3\\2/3\\-2/3\end{bmatrix}}\ \sigma_P$

II $\chi_{S,2}:\ \overrightarrow{SM'}=2\overrightarrow{SM}$

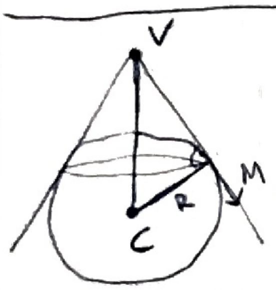
$\Rightarrow \begin{pmatrix}x'-1\\y'-3\\z'-1\end{pmatrix}=2\begin{pmatrix}x-1\\y-3\\z-1\end{pmatrix} \Rightarrow \boxed{\chi_{S,2}: \begin{bmatrix}x'\\y'\\z'\end{bmatrix}=\begin{bmatrix}2&0&0\\0&2&0\\0&0&2\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}+\begin{bmatrix}-1\\-3\\-1\end{bmatrix}}$

$$[\sigma_P\circ\chi_{S,2}]=[\sigma_P]\cdot[\chi_{S,2}]=\begin{bmatrix}-2/3 & 2/3 & 1/3 & -2/3\\ 2/3 & 1/3 & 2/3 & 2/3\\ 1/3 & 2/3 & -2/3 & -2/3\\ 0&0&0&1\end{bmatrix}\begin{bmatrix}2&0&0&-1\\0&2&0&-3\\0&0&2&-1\\0&0&0&1\end{bmatrix}=\begin{bmatrix}-4/3 & 4/3 & 2/3 & -7/3\\ 4/3 & 2/3 & 4/3 & -5/3\\ 2/3 & 4/3 & -4/3 & -7/3\\ 0&0&0&1\end{bmatrix}$$

$$\Rightarrow \boxed{\sigma_P\circ\chi_{S,2}: \begin{bmatrix}x'\\y'\\z'\end{bmatrix}=\frac{1}{3}\left(\begin{bmatrix}-4&4&2\\4&2&4\\2&4&-4\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}+\begin{bmatrix}-7\\-5\\-7\end{bmatrix}\right)}$$

5 $\mathcal{K}$ са врхом $V(1,4,6)$ и додирује $\sigma: (x-1)^2+y^2+(z-3)^2=9$.

$\mathcal{K}\cap O_{xy}=?$ $\Rightarrow C(1,0,3),\ R=3$



$M(x,y,z)\in\mathcal{K}$ произв $\Rightarrow VM=i$ је изводница

$\Rightarrow \mathcal{K}: d(C,i)=R \Leftrightarrow \frac{\|\overrightarrow{VC}\times\overrightarrow{VM}\|}{\|\overrightarrow{VM}\|}=3$

$\overrightarrow{VC}\times\overrightarrow{VM}=\begin{vmatrix} i & j & k\\ 0 & -4 & -3\\ x-1 & y-4 & z-6\end{vmatrix}=(3y-4z+12,\ -3(x-1),\ 4(x-1))$

$\Rightarrow \mathcal{K}: \frac{\sqrt{(3y-4z+12)^2+25(x-1)^2}}{\sqrt{(x-1)^2+(y-4)^2+(z-6)^2}}=3\ /^2$

$\Rightarrow \boxed{\mathcal{K}: (3y-4z+12)^2+16(x-1)^2=9(y-4)^2+9(z-6)^2}$

$\mathcal{K}\cap O_{xy} \Leftrightarrow \mathcal{K}\cap z=0$

$\Rightarrow (3y+12)^2+16(x-1)^2=9(y-4)^2+9\cdot 36$

$\cancel{9y^2}+72y+\cancel{144}+16x^2-32x+16=\cancel{9y^2}-72y+\cancel{144}+324$

$16(x-1)^2=-144y+324=-9\cdot 16y+9\cdot 36$

$16(x-1)^2=-36(4y-9)\ /:4$

$\Rightarrow \boxed{\mathcal{P}: 4(x-1)^2=-9(4y-9)}$

ПАРАБОЛА