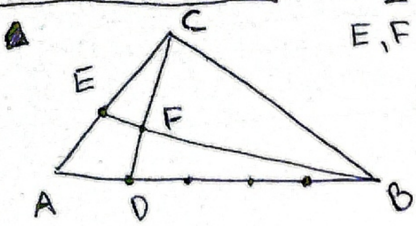
АГ - Г1 - ЈУН 2 - 2023

1 ΔABC т.д $\frac{\vec{AD}}{\vec{DB}} = \frac{1}{4}$ и $\frac{\vec{AE}}{\vec{EC}} = \frac{3}{5}$, $DC \cap BE = \{F\}$.
У ком односу F дели DC? $\vec{BF} = \alpha \vec{BD} + \beta \vec{BC}$?

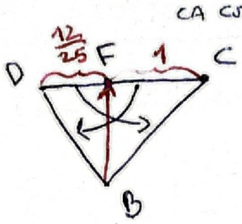
▲



ΔADC Менелај $\Longleftrightarrow$ $\frac{\vec{AB}}{\vec{BD}} \cdot \frac{\vec{DF}}{\vec{FC}} \cdot \frac{\vec{CE}}{\vec{EA}} = -1$

E, F, B колин

$\frac{\vec{AB}}{\vec{BD}}$ = $-\frac{5}{4}$ са слике, $\frac{\vec{CE}}{\vec{EA}} = 5/3$ $\Rightarrow$ $\boxed{\frac{\vec{DF}}{\vec{FC}} = \frac{12}{25}}$



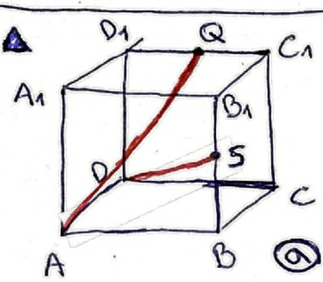
$\vec{BF} = \frac{1}{\frac{12}{25}+1}\vec{BD} + \frac{\frac{12}{25}}{\frac{12}{25}+1}\vec{BC}$

$\Rightarrow \boxed{\vec{BF} = \frac{25}{37}\vec{BD} + \frac{12}{37}\vec{BC}}$ ■

2 Коцка $ABCDA_1B_1C_1D_1$ ивице a. $S = S(BB_1)$ и $Q = S(D_1C_1)$.

ⓐ $\measuredangle(\vec{DS}, \vec{AQ}) = ?$ ⓑ $d(DS, AQ) = ?$

▲



Посматрајмо ОНБ $\vec{e_1} = \frac{\vec{AB}}{a}$, $\vec{e_2} = \frac{\vec{AD}}{a}$, $\vec{e_3} = \frac{\vec{AA_1}}{a}$

$\Rightarrow A(0,0,0)$, $Q(\frac{a}{2}, a, a)$, $D(0,a,0)$, $S(a, 0, \frac{a}{2})$

$\Rightarrow \vec{AQ} = (\frac{a}{2}, a, a)$, $\vec{DS} = (a, -a, \frac{a}{2})$, $\vec{AD} = (0, a, 0)$

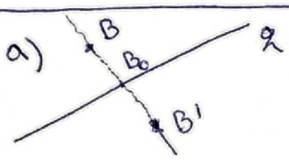
ⓐ $\vec{AQ} \circ \vec{DS} = \frac{a^2}{2} - a^2 + \frac{a^2}{2} = 0 \Rightarrow \boxed{\vec{AQ} \perp \vec{DS}}$

ⓑ $d(DS, AQ) = \frac{|[\vec{DS}, \vec{AQ}, \vec{AD}]|}{\|\vec{DS} \times \vec{AQ}\|} \overset{ⓐ}{=} \frac{\left| \begin{vmatrix} a & -a & a/2 \\ a/2 & a & a \\ 0 & a & 0 \end{vmatrix} \right|}{\|\vec{DS}\| \cdot \|\vec{AQ}\|} = \frac{|a(-1)^{2+3} \cdot (a^2 - \frac{a^2}{4})|}{(\sqrt{2a^2 + \frac{a^2}{4}})^2} = \frac{\frac{3a^3}{4}}{\frac{9a^2}{4}} = \boxed{\frac{a}{3}}$ ■

3 $A(-3,-3)$, $B(0,-3)$, $p: x - y - 2 = 0$, $q: x + y + 2 = 0$

ⓐ B' сим. B у одн. q ⓑ крива садржи A и B а p и q осе сим.

▲ а)

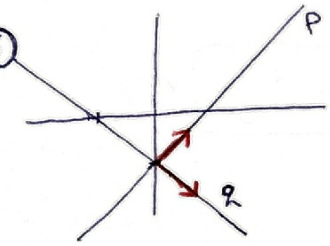


1) n т.д $B \in n$, $n \perp q$ $\Rightarrow n: \begin{matrix} x = t \\ y = t - 3 \end{matrix}$, $t \in \mathbb{R}$

2) $B_0 = n \cap q$: $t + t - 3 + 2 = 0 \Rightarrow t = 1/2 \Rightarrow B_0(\frac{1}{2}, -\frac{5}{2})$

3) $B(0,-3)$, $B_0(\frac{1}{2}, -\frac{5}{2}) \Rightarrow \boxed{B'(1, -2)}$

ⓑ



Бирамо нови коорд систем т.д q буде x' оса, p буде y'-оса и $p \cap q = O'$ нови коорд почетак

$p \cap q = O'(0,-2)$, $\vec{f_1} = (\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}})$, $\vec{f_2} = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$

$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix} + \begin{bmatrix} 0 \\ -2 \end{bmatrix} \Rightarrow \boxed{\begin{bmatrix} x' \\ y' \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} \sqrt{2} \\ -\sqrt{2} \end{bmatrix}}$

$\Rightarrow \mathcal{K}': \frac{x'^2}{a^2} + \mu \frac{y'^2}{b^2} = 1$, $\mu \in \{\pm 1\}$, $A(-3,-3) \longmapsto A'(-\sqrt{2}, -2\sqrt{2})$, $B \longmapsto B'(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}})$

1) $A' \in \mathcal{K}' \Rightarrow \frac{2}{a^2} + \mu \cdot \frac{8}{b^2} = 1 \Leftrightarrow 2b^2 + 8\mu a^2 = a^2b^2$ $/ \cdot 2$ $\quad -3b^2 - 15\mu a^2 = 0$

2) $B' \in \mathcal{K}' \Rightarrow \frac{1}{2a^2} + \mu \cdot \frac{1}{2b^2} = 1 \Leftrightarrow b^2 + \mu a^2 = 2a^2b^2$ $+$ $\quad b^2 = -5\mu a^2 \Rightarrow \boxed{\mu = -1}$

$\Rightarrow$ $\boxed{\text{хипербола } b^2 = 5a^2}$

$5a^2 - a^2 = 10a^4$ $/ : a^2$

$\boxed{a^2 = \frac{2}{5} \Rightarrow b^2 = 2}$

$$\Rightarrow \ \mathcal{X}': \frac{5x'^2}{2} - \frac{y'^2}{2} = 1 \iff \boxed{5x'^2 - y'^2 = 2}$$

$$\Rightarrow \ \mathcal{X}: 5\left(\frac{1}{\sqrt{2}}(x-y-2)\right)^2 - \left(\frac{1}{\sqrt{2}}(x+y+2)\right)^2 = 2 \quad / \cdot 2$$

$$\boxed{\mathcal{X}: 5(x-y-2)^2 - (x+y+2)^2 = 4} \iff \boxed{\begin{aligned} \mathcal{X}: x^2 - 3xy + y^2 \\ -6x + 4y + 3 = 0 \end{aligned}}$$ ■

4 РОТАЦИЈА ОКО $S(1,2)$ ЗА $\varphi = \pi/3$? СЛИКА $k: (x-4)^2 + (y-2)^2 = 9$

$\cos \pi/3 = 1/2$

▲ $R_{S,\varphi} = I_{\vec{OS}} \circ R_{O,\varphi} \circ I_{\vec{SO}}$, $\vec{SO} = (-1,-2)$, $\vec{OS} = (1,2)$, $\sin \pi/3 = \sqrt{3}/2$

$$[R_{S,\varphi}] = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \left(\begin{bmatrix} 1/2 & -\sqrt{3}/2 & 0 \\ \sqrt{3}/2 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/2 & -\sqrt{3}/2 & -\frac{1}{2} + \sqrt{3} \\ \sqrt{3}/2 & 1/2 & -\sqrt{3}/2 - 1 \\ 0 & 0 & 1 \end{bmatrix}$$

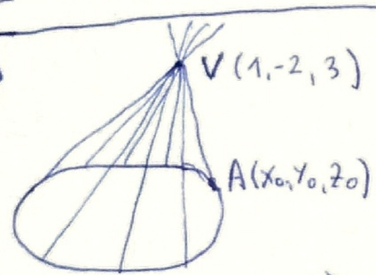
$$= \begin{bmatrix} 1/2 & -\sqrt{3}/2 & 1/2 + \sqrt{3} \\ \sqrt{3}/2 & 1/2 & 1 - \sqrt{3}/2 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \boxed{R_{S,\frac{\pi}{3}}: \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1/2 + \sqrt{3} \\ 1 - \sqrt{3}/2 \end{bmatrix}}$$

2) k се слика у k' истог полупречника, $C(4,2) \longmapsto C'\left(\frac{5}{2}, \frac{3\sqrt{3}}{2} + 2\right)$

$$\Rightarrow \boxed{k': \left(x - \frac{5}{2}\right)^2 + \left(y - 2 - \frac{3\sqrt{3}}{2}\right)^2 = 9}$$ ■

5 КОНУС СА ВРХОМ $V(1,-2,3)$ КОЈИ САДРЖИ $\mathcal{E}: 2x^2 + 3y^2 = 1, \ z = 0$.

▲



Нека је $A(x_0, y_0, z_0) \in \mathcal{E}$ произв.

$\Rightarrow$ произвољна изводница конуса је права VA

$$\Rightarrow \ i: \frac{x-1}{x_0-1} = \frac{y+2}{y_0+2} = \frac{z-3}{z_0-3} = t$$

$$\Rightarrow \ x_0 = \frac{x-1}{t} + 1 \overset{(*)}{=} \frac{3x-3}{3-z} + 1 = \frac{3x-z}{3-z}$$

$$y_0 = \frac{y+2}{t} - 2 = \frac{3y+6}{3-z} - 2 = \frac{3y+2z}{3-z}$$

$$z_0 = \frac{z-3}{t} + 3 = \frac{3z-9}{3-z} + 3 = 0$$

1) $A \in \mathcal{E} \Rightarrow z_0 = 0 \Rightarrow \frac{z-3}{t} = -3 \Rightarrow z - 3 = -3t \Rightarrow \boxed{t = \frac{3-z}{3}}$ $(*)$

2) $A \in \mathcal{E} \Rightarrow 2x_0^2 + 3y_0^2 = 1$

$$\Rightarrow 2\left(\frac{3x-z}{3-z}\right)^2 + 3\left(\frac{3y+2z}{3-z}\right)^2 = 1 \quad / (z-3)^2$$

$$\boxed{\mathcal{K}: 2(3x-z)^2 + 3(3y+2z)^2 = (z-3)^2}$$ ■